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Milnor number equals Tjurina number for functions on space curves. (English summary)
J. London Math. Soc. (2) **63** (2001), *no. 1,* 177–187.

Let X be a reduced one-dimensional germ in $(\mathbf{C}^m, 0)$, and let $f \in \mathcal{O}_X$ be a function on X. Then f defines a map $f\colon X \to S$, where S is the germ of a smooth curve. Assume that the function f is non-constant on each branch of X. Then $\mathcal{O}_X$ is a finite and free $\mathcal{O}_S$-module via f. The Milnor number of the function f is defined as follows: $\mu(f) = \dim_{\mathbf{C}} \omega_X/\mathcal{C}(df \wedge \mathcal{O}_X)$, where ω_X is the Grothendieck dualizing module, and $\mathcal{C}\colon \Omega^1_X \to \omega_X$ is the fundamental class map. Let T^i_X and $T^i_{X/S}$, $i = 0, 1, 2$, be the absolute and relative cotangent cohomology modules, respectively, in the sense of S. Lichtenbaum and M. Schlessinger [Trans. Amer. Math. Soc. **128** (1967), 41–70; MR0209339 (35 #237)]. Let $\tau(f) = \dim_{\mathbf{C}} T^1_{X/S}$ be the Tyurina number of the function f. The authors prove that if $T^2_X = 0$ and X is smoothable then $\mu(f) = \tau(f)$. In particular, this result implies that the discriminant of the minimal versal deformation of the map f is a free Saito divisor [K. Saito, J. Fac. Sci. Univ. Tokyo Sect. IA Math. **27** (1980), no. 2, 265–291; MR0586450 (83h:32023)]. The authors underline that the equality of Milnor and Tyurina numbers for functions on space curve singularities $X \subset (\mathbf{C}^3, 0)$ was conjectured by V. V. Goryunov [see J. London Math. Soc. (2) **61** (2000), no. 3, 807–822 MR1766106 (2002f:58068)]. They also remark that a similar statement is true in more general cases, e.g., for any unobstructed smoothable curve X of arbitrary embedding dimension.

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